1.7 Inequalities

Starter Questions

Find the set of values of which satisfy the following inequalities:

$$5x+9\geq x+20$$

$$3(x-5)>5-2(x-8)$$

$$2x2-3x-5\geq 0$$

$$12+4x>x2$$

Solve the inequalities

a)

b)

Hence, find the values of which satisfy both:

and

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1.7 Inequalities

Starter Questions

Find the set of values of which satisfy the following inequalities:

$$5x+9 \geq x+20 \qquad x \geq \frac{11}{4}$$

$$3(x-5)>5-2(x-8) \qquad x>\frac{36}{5}$$

$$2x2-3x-5 \geq 0 \qquad x \leq -1,\ x \geq \frac{5}{2}$$

$$12+4x>x2 \qquad -2<x<6$$

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1.7 Inequalities

tarter Questions

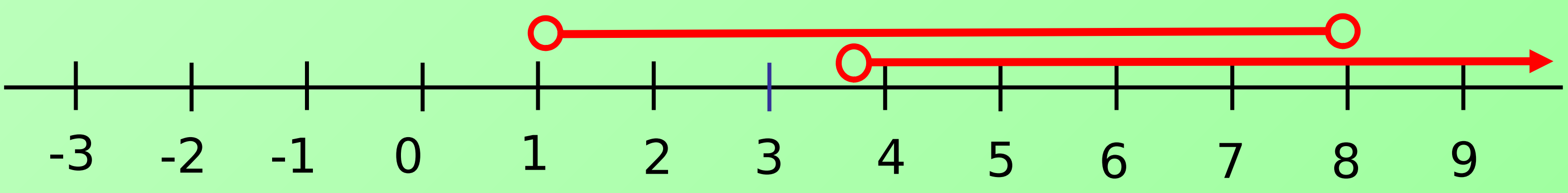
Solve the inequalities

a)

b)

Hence, find the values of which satisfy both:

and



$$\frac{11}{3} < x < 8$$

B Algebra and functions

B5

Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions.

Express solutions through correct use of 'and' and 'or', or through set notation.

Represent linear and quadratic inequalities such as $y > x + 1$ and $y > ax^2 + bx + c$ graphically.

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- give the range of values which satisfy more than one inequality
- illustrate regions on sketched graphs, defined by inequalities
- define algebraically inequalities that are given graphically.

B Algebra and functions

Notes

- Dotted/dashed lines or curves will be used to indicate strict inequalities.
- Overarching theme 1.3 is of particular relevance here. Students are required to demonstrate an understanding of and use the notation (language and symbols) associated with set theory (as set out in Appendix A of the specification). Students may be required to apply this notation to the solutions of inequalities.
- There are many ways of representing solution sets using set notation. Typically the variable used is the same as that used in the question, but any letter could be used. We would advise using x or y whenever possible.
- Students are expected to understand notation such as:
    - $\{x : x < 2\}$ or $(-\infty, 2)$
    - $\{x : x \le -1\} \cup \{x : x \ge 2\}$ or $(-\infty, -1] \cup [2, \infty)$
- There is no need to state $x \in \mathbb{R}$, because we assume we are using real numbers.
- If a question requires an answer to be written in set notation we would accept any mathematically correct notation; we would accept the word 'or' instead of '$\cup$' Our intention is always to reward correct mathematics.
- Questions will always say how a required region should be indicated eg shade and label the region *R*.

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1.7 Inequalities

Set notation	Meaning
	The set with elements
	The set of all such that…
	The closed interval
	The interval
	The interval
	The open interval
	Union (or)
	Intersection (and)
	Infinity
	Negative infinity

1.7 Inequalities

Students are expected to understand notation such as:

- $\{x : x < 2\}$ or $(-\infty, 2)$
- $\{x : x \le -1\} \cup \{x : x \ge 2\}$ or $(-\infty, -1] \cup [2, \infty)$

$\square < \mathbf{2}$

or

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1.7 Inequalities

Example 1

Rewrite the answers to the starter questions in set notation.

$5x+9\geq x+20$ $\quad x\geq\frac{11}{4}$ $\quad \left\{x: x\geq\frac{11}{4}\right\}$

$3(x-5)>5-2(x-8)$ $\quad x>\frac{36}{5}$ $\quad \left\{x: x>\frac{36}{5}\right\}$

$2x2-3x-5\geq 0$ $\quad x\leq -1, x\geq\frac{5}{2}$

$\{x: x\leq -1\}\cup\left\{x: x\geq\frac{5}{2}\right\}$

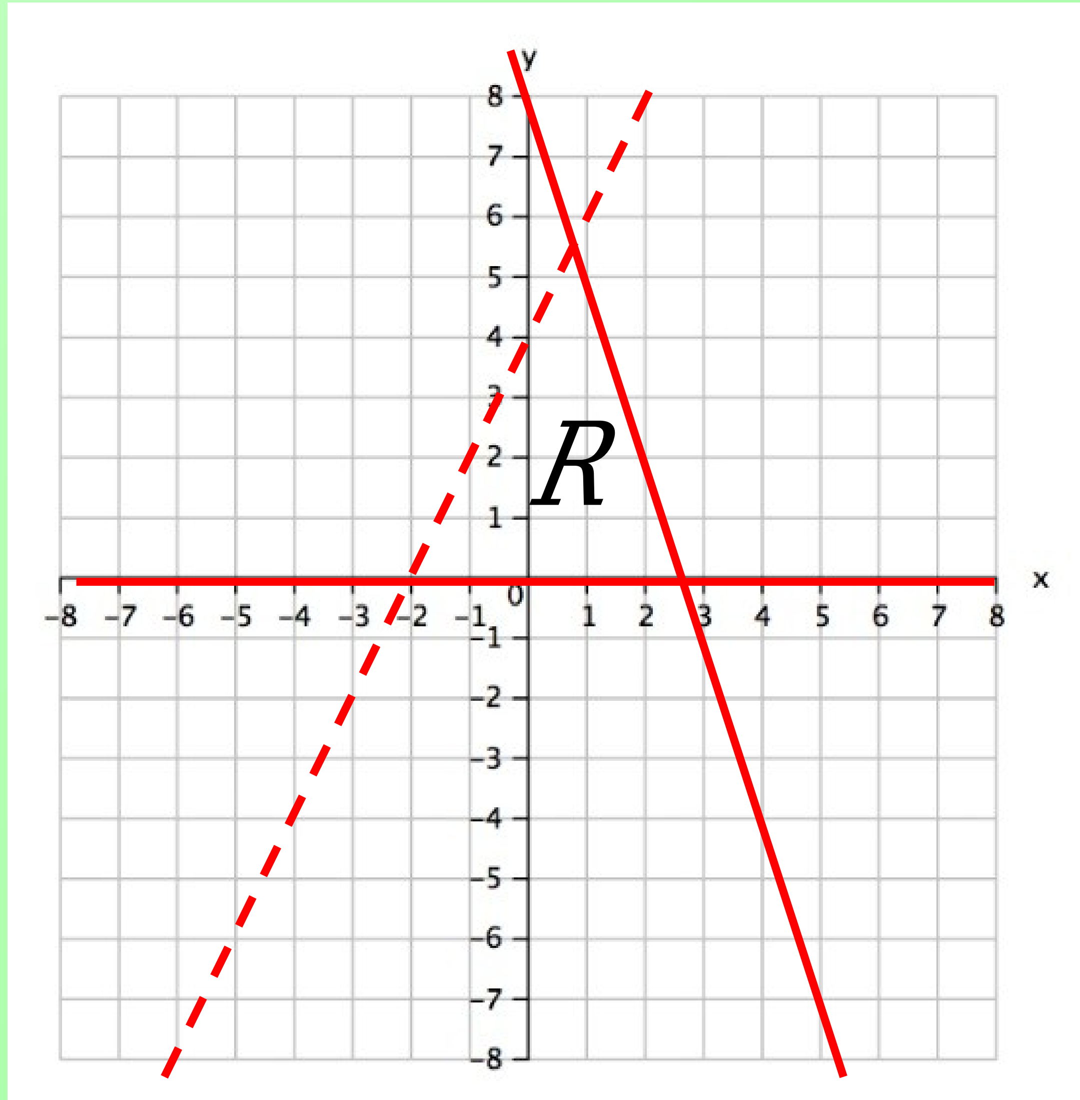
$12+4x>x2$

$-2<x<6$ $\quad \{x: -2<x<6\}$

1.7 Inequalities

Example 2

Label the region, , that satisfies the inequalities:



For or lines should be dashed.
Solid lines indicate or

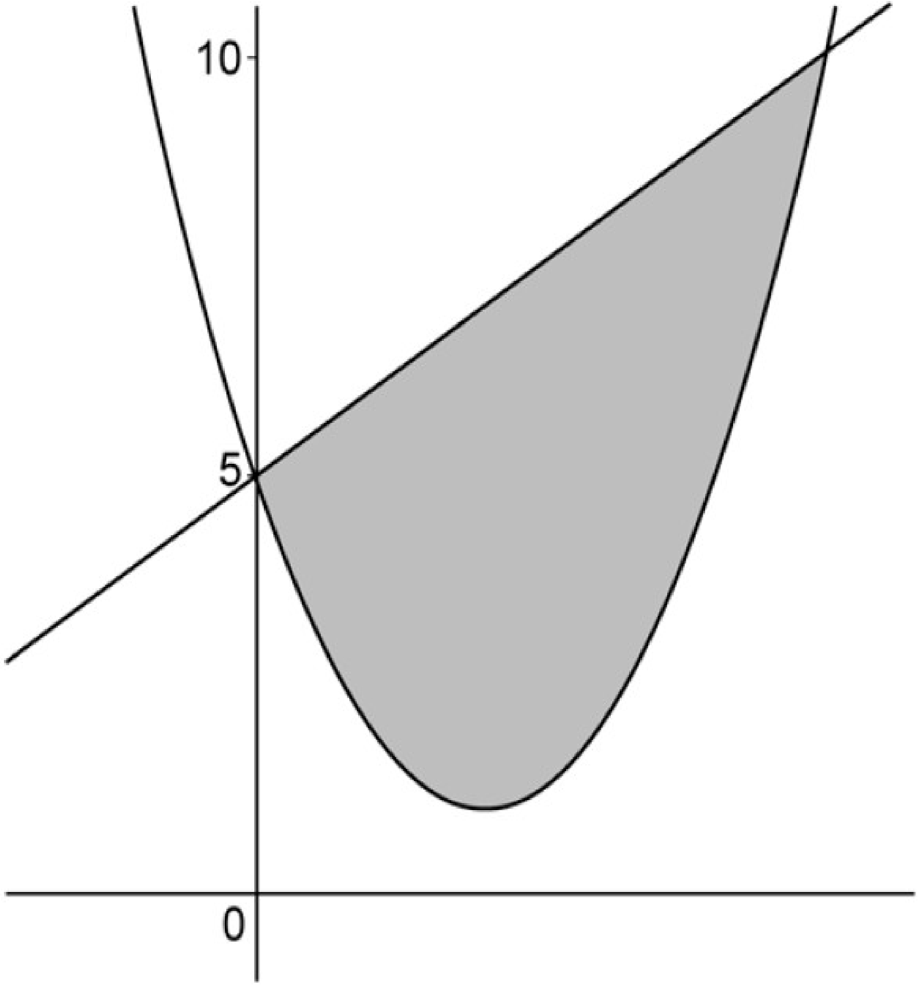
1.7 Inequalities

Example 1

State the inequalities which define the shaded region.

The diagram shows the graphs of $y = x + 5$ and $y = x^2 - 4x + 5$

$$\boxed{\begin{array}{c} y \le x + 5 \\ \text{and} \\ y \ge x^2 - 4x + 5 \end{array}}$$



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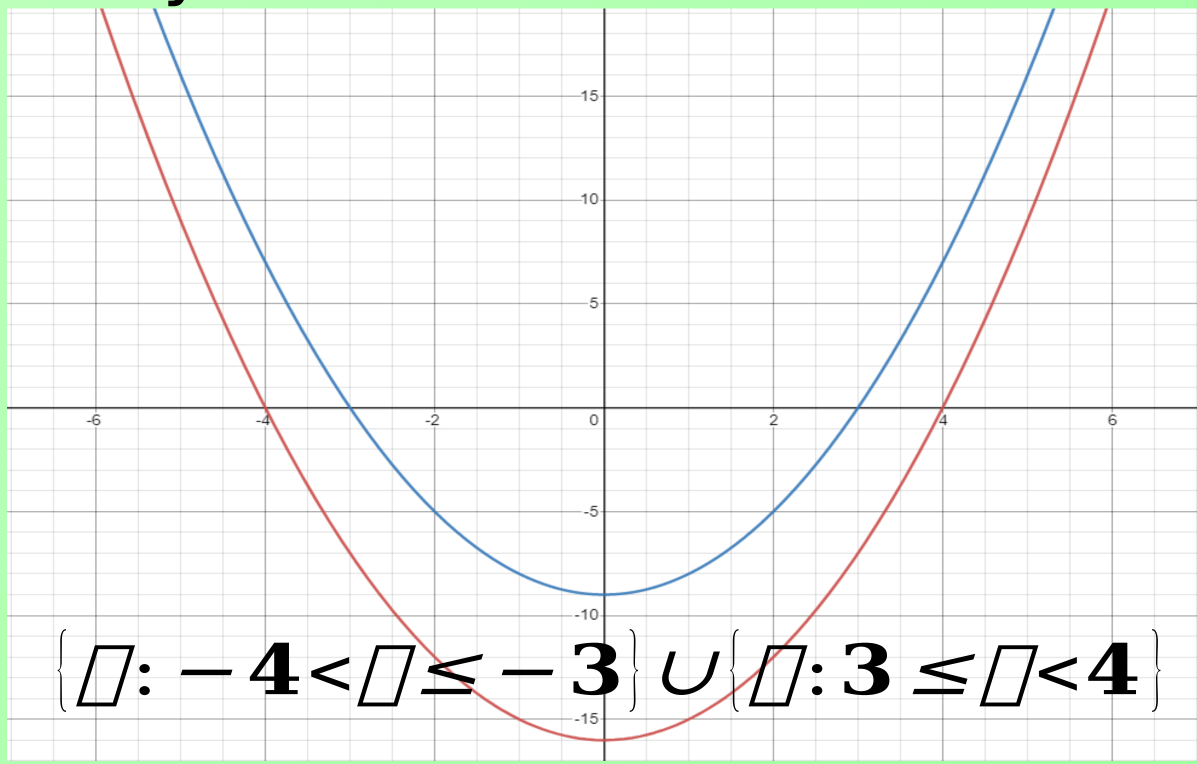
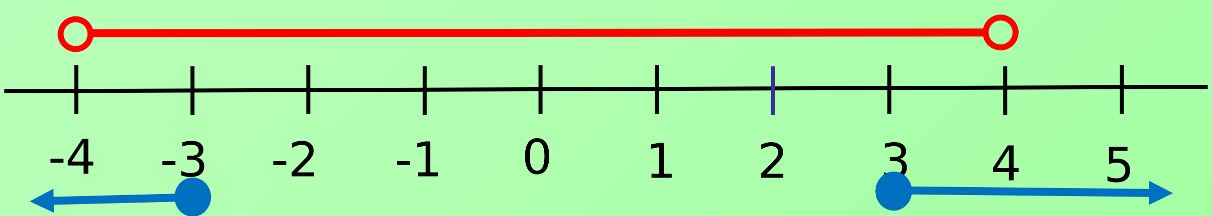
1.7 Inequalities

Example 4

Solve simultaneously with



$$\{x: -4 < x \le -3\} \cup \{x: 3 \le x < 4\}$$



A Proof

A1	Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion. Disproof by counter example.

Assessed at AS and A-level

Teaching guidance

Students should be able to:

- set out a clear proof with the correct use of symbols, such as $=$, $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$, $\equiv$, $\therefore$, $\because$
- understand that considering examples can be useful in looking for structure, but this does not constitute a proof.

A Proof

A1	Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion. Disproof by counter example.

Notes

- At A-level 25% (20% at AS) of the assessment material must come from Assessment Objective 2 (reason, interpret and communicate mathematically). A focus on clear reasoning, mathematical argument and proof using precise mathematical language and notation should underpin the teaching of this specification. Students should become familiar with the mathematical notation found in Appendix A of the specification.
- It will not be essential to use any particular notation when writing answers to exam questions, but some questions could assess understanding of this notation.
- Students should understand the sets of numbers $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

1.1 Argument & Proof

The idea of proof

A statement that has been proven is called a **theorem**, e.g. Pythagoras' Theorem.

A statement that is yet to be proven is called a **conjecture**.

A **proof** is a logical and structured argument to show that a mathematical statement is always true. It should be structured so that it follows a logical series of steps.

1.1 Argument & Proof

Equations & Identities

is an **equation**. An equation is only true for **some** values of , in this case, .

is an **identity**. An identity is true for **all** values of .

Some rules only apply to identities, e.g. if two polynomials are identically equal the coefficients of corresponding variables must be the same.

1.1 Argument & Proof

Example 1

Find the values of and.

Coefficient of :
Coefficient of :
Constant term:

1.1 Argument & Proof

Proving Identities

Identities are **not** equations so we cannot treat them as such.

To prove an identity:

1. Begin with one side of the identity (usually the left-hand side)
2. Manipulate it algebraically to get the expression on the other side
3. Show every step of working

1.1 Argument & Proof

Example 2 Prove that $(x + y)^2 - (x - y)^2 \equiv 4xy$.

LHS:

RHS as required

1.1 Argument & Proof

Proof with quadratics

The main two proofs with quadratics are:

1. Those involving how many roots a quadratic has which involves discriminant analysis.
2. Proofs involving inequalities.

1.1 Argument & Proof

Example

Prove that $3x^2 + 12x + 16 > 0$ for all values of x.

as any number squared is

This proves that